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# ON HYPERSONIC FLOW PAST OF A LIFT AIRFOIL*

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The asymptotic solution of the Navier-Stokes equation is studied at large distance past of a lift airfoil of finite dimensions. The flow field is divided in three regions: the external flow, the laminar trail and the subtrail. The main attention is given to singularities associated with the lift force. It is shown that the subtrail, generated only in the presence of lift, has the contour of an oscillating cord, and the gas particles in every transverse plane inside the subtrail are defined only by the radial components of the velocity vector, if the coordinate origin is selected in a particular way.

**1. Analysis of flow in the external region and in the laminar trail.** Let us consider a steady hypersonic flow at large distance from the airfoil. Let $\rho_\infty$ be the gas density in the oncoming stream, $U_\infty$ its velocity along the $x$ axis of a cylindrical system of coordinates $(x, r, \varphi)$. We assume that in the unperturbed stream the pressure is zero. We consider the gas perfect, i.e. conforming to the Clapeyron equation of state, with the two specific heats $c_p$ and $c_v$ assumed constant; we denote their ratio by $\varkappa$ and assume $1 < \varkappa < 2$. The dependence of viscosity coefficients $\lambda_1$ and $\lambda_2$, and of thermal conductivity $k$ on specific enthalpy $w$ are assumed linear: $\lambda_1 = \lambda_{10}w$, $\lambda_2 = \lambda_{20}w$, $k = k_0w$. The Prandtl number is denoted by $N_{\text{Pr}} = c_p\lambda_{10}/k_0$. The independent variables, as well as the unknown functions are conveniently specified as dimensionless quantities, using as the basic scale $\rho_\infty$, $U_\infty$, $\lambda_{10}$.

The principal terms of the asymptotic solution of the Navier-Stokes equations at considerable distances past of the lift airfoil in an axisymmetric hypersonic flow were obtained by V.V. Sychev /1/. Perturbations of the axisymmetric solution, which enabled the description of the flow with a finite lift force, were studied in /2,3/. The derived in /1,3/ scheme of the stream has two essentially different regions: the external and the laminar trail. In the external region it is possible to neglect the effects of viscosity and thermal conductivity. The external region is separated from the oncoming stream by the curved shock wave whose structure was investigated in /4/. As shown in /3/, parameters of the hypersonic viscous stream behind the lift airfoil can be obtained in the external region from the solution of the Cauchy problem for Euler's equations. For this it is necessary to specify the Rankin-Hugoniot conditions behind the shock wave whose form with $x \to \infty$ is given by the equation

$$r_s = (bx)^{1/2}(1 + b_y x^{-1/2} \ln x \cos \varphi + \ldots) \tag{1.1}$$

where constants $b$ and $b_y$ are determined by the drag and lift forces. The first term of expansion (1.1) depends only on drag. The solution generated by it is well known as the solution of the problem of strong cord explosion defined by L.I. Sedov /5,6/. The second term of expansion (1.1) is determined by the lift force. Its solution was studied in /2/. It consists of two terms. Let us write the form of solution for transverse velocities

$$v_r = \frac{1}{\varkappa+1}\left(\frac{b}{x}\right)^{1/2}\{v_{r11}(\xi) + b_y x^{-1/2}[\ln x v_{r12}(\xi) + v_{r13}(\xi)]\cos\varphi + \ldots\} \tag{1.2}$$

$$v_\varphi = \frac{1}{\varkappa+1}\frac{b^{1/2}}{x} b_y\{[\ln x v_{\varphi 12}(\xi) + v_{\varphi 13}(\xi)]\sin\varphi + \ldots\},\quad \xi = \frac{r}{(bx)^{1/2}}$$

and analyze the direction field defined by the velocities (1.2). For this we introduce in the transverse plane a Cartesian system of coordinates $(y, z)$, with $y$ directed along the line of lift force $F_y$. As follows from /2/, the flow field in the transverse plane defined by functions $v_{r11}$, $v_{r12}$, $v_{\varphi 12}$ possesses a central symmetry relative to point $(y = -b^{1/2}b_y \ln x,\ z = 0)$. We shall consider the direction field generated by function $v_{r13}$ and $v_{\varphi 13}$. It is precisely with these functions that the presence of vortices in perturbation motion is associated. For

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convenience of presentation we do not show the actual direction field, but the lines the tangents to which represent the indicated field. For it is sufficient to integrate the equation $dy/dz = v_{y13}/v_{z13}$ at constant $x$. However, it is more convenient to have the last equation of the system in the form differentiable with respect to the parameter $t$

$$dy/dt = v_{r13}\cos^2\varphi - v_{\varphi 13}\sin^2\varphi,\ dz/dt = (v_{r13} + v_{\varphi 13})\cos\varphi\sin\varphi \tag{1.3}$$

Drawing the integral curves (1.3) through points lying on the semi-axis $y = 0, z > 0$, we obtain the picture represented in Fig.1. In the half-plane $z < 0$ the integral curves (1.3) are symmetric about the straight line $z = 0$ to curves in Fig.1. The shock wave is represented there by the dash line. Here $y_1 = yb^{-1/2}x^{-1/2}$, $z_1 = zb^{-1/2}x^{-1/2}$ and in calculation $\varkappa = 1.4$ was assumed. However, for other values of $\varkappa$ in the range $1 < \varkappa < 2$ the qualitative pattern of integral curves is not altered. It is seen from Fig.1 that in the transverse plane exists a system of local vortex zones. The first of these, adjoining the shock wave, $y_1^2 + z_1^2 = 1$, is formed by open lines that begin and end on the shock wave. The remaining zones are formed by closed curves. These zones are between themselves separated by circles whose radius is determined by the equation $v_{r13}(\xi) = 0$, their centers lie on the $y = 0$ axis, and the coordinate $z_1$ is determined by points $\xi$ for which $v_{\varphi 13}(\xi) = 0$. The first three points that determine the centers are $\xi = 1;\ 0.533;\ 0.11$.

Using the explicit expression /2/ for terms with index 12

$$v_{r12} = -dv_{r11}/d\xi,\ v_{\varphi 12} = v_{r11}/\xi$$

we calculate the longitudinal component of the vortex vector. With the accuracy to terms given in (1.2) we have

$$\omega_x = \frac{1}{\varkappa + 1} b_y x^{-3/2} A_1 \sin\varphi,\quad A_1 = \frac{dv_{\varphi 13}}{d\xi} + \frac{v_{r13} + v_{\varphi 13}}{\xi} \tag{1.4}$$

The component $\omega_x$ in the considered approximation is determined by the lift force; its intensity, when approaching the coordinate origin has an oscillating character with rapidly increasing amplitude. The curve of dependence of quantity $A_1$ on $\xi$ when $\varkappa = 1.4$ is represented in Fig.2, which implies that $A_1$ has a singularity at $\xi = 0$ and the points of intermediate maxima $|\omega_x|$ behind the shock wave do not coincide with the vortex centers (the first two points of maximum are $\xi = 0.20;\ 0.04$).

Let us carry out a similar analysis for the laminar trail. According to /3/ the transverse components of the velocity vector are of the form

$$\begin{aligned} v_r &= \frac{1}{\varkappa + 1} b^{1/4} x^{-\varkappa/(\varkappa+1)} \{v_{r21}(\zeta) + b_y x^{-(2-\varkappa)/2(\varkappa+1)} \times \\ &\quad [v_{r2c}(\zeta)\cos(k_3 \ln x) + v_{r2s}(\zeta)\sin(k_3 \ln x)]\cos\varphi + \ldots\} \\ \zeta &= rb^{-1/2}x^{-1/(\varkappa+1)} \\ v_\varphi &= \frac{1}{\varkappa + 1} b^{1/2} b_y x^{-(2+\varkappa)/2(\varkappa+1)} [v_{\varphi 2c}(\zeta)\cos(k_3 \ln x) + \\ &\quad v_{\varphi 2s}(\zeta)\sin(k_3 \ln x)]\sin\varphi + \ldots,\quad k_3 = \frac{\varkappa - 1}{2(\varkappa + 1)}\sqrt{\frac{3 - \varkappa}{\varkappa - 1}} \end{aligned} \tag{1.5}$$

The principal term in the expansion of radial velocity $v_{r21}$ defines the isotropic spreading independent of angle $\varphi$ which is determined solely by the drag force. The addition to it depends, as in the external region, on angle $\varphi$; the dependence of variable $x$ is more complicated than in (1.2).

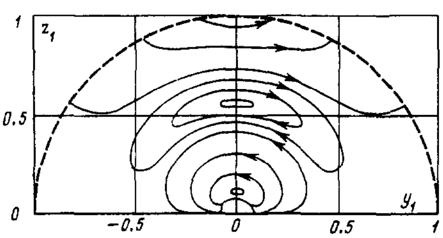

Fig.1

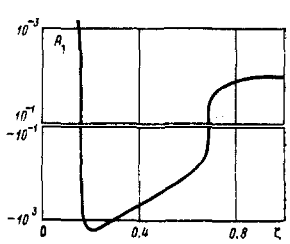

Fig.2

Here, in the study of the direction field produced by perturbations due to the lift force it is necessary to consider two systems of equations in contrast to the external region, where one can restrict the analysis to a single equation. Similarly to (1.3), we use parameter $t$ and obtain

$$\frac{dy}{dt} = v_{r2c}\cos^2\varphi - v_{\varphi 2c}\sin^2\varphi, \quad \frac{dz}{dt} = (v_{r2c} + v_{\varphi 2c})\cos\varphi\sin\varphi \tag{1.6}$$

$$\frac{dy}{dt} = v_{r2s}\cos^2\varphi - v_{\varphi 2s}\sin^2\varphi, \quad \frac{dz}{dt} = (v_{r2s} + v_{\varphi 2s})\cos\varphi\sin\varphi \tag{1.7}$$

The integrals of system of Eqs.(1.6) are linked with the direction field in these cross sections $x = \mathrm{const}$ for which $\cos(k_3 \ln x) = 1$ is satisfied; similarly the integrals of system (1.7) define the direction field there where $\sin(k_3 \ln x) = 1$. At intermediate points $x$ the solutions of both system must be multiplied by $\cos(k_3 \ln x)$ and $\sin(k_3 \ln x)$ and summed up. Drawing the integral curves of systems (1.6) and (1.7) from points lying on the semiaxis $y = 0, z > 0$, we obtain the patterns shown in Figs.3 and 4, respectively. For the half-plane $z < 0$ the integral curves of (1.6) and (1.7) are symmetric about the straight line $z = 0$ to curves in Figs. 3 and 4. Here $y_2 = yb^{-1/2}x^{-1/(\varkappa+1)}, z_2 = zb^{-1/2}x^{-1/(\varkappa+1)}$ and in calculations it was assumed that $\varkappa = 1.4$; $N_{\mathrm{Pr}} = 0.75$; $\lambda_{20}/\lambda_{10} = 0.1$. The pattern in Fig.3 is similar to the inner part shown in Fig.1, while the pattern in Fig.4 is substantially different. In the inner part of Fig.4 there is a vortex with its center on the axis $y = 0$, and in the external part we have on the axis $z = 0$ a source and a sink.

Let us determine the longitudinal component of the vortex vector. With the accuracy to terms appearing in (1.5), we have

$$\omega_x = \frac{1}{\varkappa+1} b_y x^{-1/2-(2\varkappa-1)/2(\varkappa+1)} [A_2\cos(k_3 \ln x) + A_3 \times \sin(k_3 \ln x)]\sin\varphi$$

$$A_2 = \frac{dv_{\varphi 2c}}{d\zeta} + \frac{v_{r2c} + v_{\varphi 2c}}{\zeta}, \quad A_3 = \frac{dv_{\varphi 2s}}{d\zeta} + \frac{v_{r2s} + v_{\varphi 2s}}{\zeta}$$

The curves of dependence of $A_2$ and $A_3$ on $\zeta$ are shown in Fig.5. The component $\omega_x$ in the considered approximation is linked to the lift force. It is regular throughout the transverse plane, as it moves away from the coordinate origin it has an oscillating character, and its intensity rapidly diminishes. The maximum points $|\omega_x|$ do not coincide with the centers of vortices.

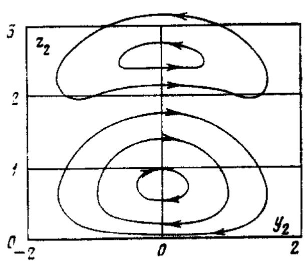


Fig.3

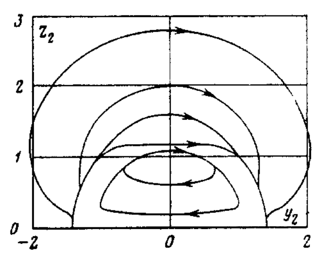


Fig.4

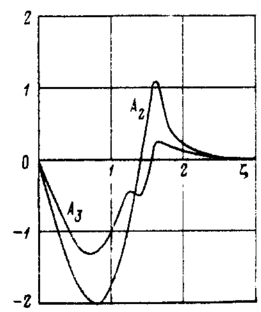


Fig.5

## 2. The flow in the region of subtrail.

Although the asymptotics of functions (1.5) do not have singularities as $\zeta \to 0$, nevertheless for small $\zeta$ in (1.5) the sequence order of terms may change, since function $v_{r21}$ is proportional to $\zeta$, while $v_{r2c}$ and $v_{r2s}$ approach to constant quantities. The possibility of formal change of sequence of terms in the considered here problem requires additional investigation, since the axis $r = 0\ (\zeta = 0)$ is singular for the input Navier–Stokes equations. For the study of flow in the neighborhood of $\zeta \to 0$ from the terms comparison of the first and second approximation with respect to the order of smallness (according to the power of the lengthwise variable $x$) we introduce new self-similar variable $\eta$ and the characteristic transverse variable for the new region of subtrail, namely

$$\eta = r/(b^{1/2}x^{\varkappa/2(\varkappa+1)})$$

Solution in the region $\eta = 0(1)$ on the basis of the analysis given in /3/ of asymptotics as $\zeta \to 0$, we seek functions of the first and second approximation in the form

$$v_r = \frac{1}{\varkappa+1} b^{1/2}x^{-(2+\varkappa)/2(\varkappa+1)} v_{r31}(\eta, \varphi, x) + \ldots \tag{2.1}$$

$$v_\varphi = \frac{1}{\varkappa+1} b^{1/2}x^{-(2+\varkappa)/2(\varkappa+1)} v_{\varphi 31}(\eta, \varphi, x) + \ldots$$

$$v_x = 1 - \frac{1}{2(\varkappa+1)} b^{1/2} x^{-\varkappa/(\varkappa+1)} v_{x31}(\eta, \varphi, x) + \ldots$$
$$\rho = \frac{\varkappa+1}{\varkappa-1} x^{-1/(\varkappa+1)} \rho_{31}(\eta, \varphi, x) + \ldots$$
$$p = \frac{1}{2(\varkappa+1)} \frac{b}{x} p_{31}(\eta, \varphi, x) + O(x^{-1-\varkappa/(\varkappa+1)})$$
$$w = \frac{\varkappa}{2(\varkappa+1)^2} b x^{-\varkappa/(\varkappa+1)} w_{31}(\eta, \varphi, x) + \ldots$$

The limit conditions at $\eta \to \infty$ for the newly introduced functions are found from the asymptotics of the trail functions as $\zeta \to 0$. As the result, we have

$$v_{r31} \to \frac{1}{2}\eta - B\cos\varphi + \ldots, \quad v_{\varphi 31} \to B\sin\varphi + \ldots \tag{2.2}$$
$$B = b_y\,[C_1 \cos(k_3 \ln x) + C_2 \sin(k_3 \ln x)]$$
$$v_{x31} \to v_{x21}(0), \quad \rho_{31} \to \rho_{21}(0), \quad p_{31} \to p_{21}(0), \quad \omega_{31} \to \omega_{21}(0)$$

Using the results of calculations in /3/ for $\varkappa = 1.4$; $N_{Pr} = 0.75$; $\lambda_{20}/\lambda_{10} = 0.1$, we have $C_1 = 0.714$; $C_2 = 0.782$; $v_{x21}(0) = 0.449$; $\rho_{21}(0) = 0.231$; $p_{21}(0) = 0.373$; $w_{21}(0) = 1.571$. Let us, also, assume that the dependence on $x$ of unknown functions is "weak", for instance, for any $\alpha > 0$ we have for $v_{r31}$

$$\frac{v_{r31}}{x^\alpha} \to 0, \quad x^{1-\alpha}\frac{\partial v_{r31}}{\partial x} \to 0, \quad x^{2-\alpha}\frac{\partial^2 v_{r31}}{\partial x^2} \to 0$$

as $x \to \infty$. In addition to conditions (2.2) at $\eta \to \infty$, we require that as $\eta \to 0$ the unknown functions be bounded, have continuous second derivatives with respect to all arguments, and were periodic of period $2\pi$ with respect to angle $\varphi$. Substituting expansions (2.1) into the system of Navier-Stokes equations and equating terms of like powers of $x$, we obtain a system of differential equations in partial derivatives.

Projections of the equations of motion on the $r$ and $\varphi$ axes determine the pressure

$$\partial p_{31}/\partial\eta = 0, \; \partial p_{31}/\partial\varphi = 0$$

which with allowance for the limit condition (2.2) yields $p_{31} = p_{21}(0)$.

The equation of heat transfer yields

$$\frac{\partial}{\partial\eta}\left(w_{31}\frac{\partial w_{31}}{\partial\eta}\right) + \frac{1}{\eta} w_{31}\frac{\partial w_{31}}{\partial\eta} + \frac{1}{\eta^2}\frac{\partial}{\partial\varphi}\left(w_{31}\frac{\partial w_{31}}{\partial\varphi}\right) = 0 \tag{2.3}$$

Passing in Eq.(2.3) to the new independent variable $\eta_1 = \ln\eta$ and the new function $w_{311} = w_{31}^2$, we arrive at the Laplace equation

$$\frac{\partial^2 w_{311}}{\partial\eta_1^2} + \frac{\partial^2 w_{311}}{\partial\varphi^2} = 0$$

Using the conditions imposed on $w_{31}$ we find that function $w_{311}$ must be free of singularities in the whole plane $\eta_1, \varphi$. Then, on the basis of the Liouville theorem on harmonic functions, we conclude that $w_{311}$ is independent of $\eta_1$ and $\varphi$. On the other hand, using the limit condition (2.2) as $\eta_1 \to \infty$, we obtain that $w_{311}$ is also independent of variable $x$. Reverting to function $w_{31}$ we have $w_{31} = w_{21}(0)$. The equation of state allows from known $w_{31}$ and $p_{31}$ to determine $\rho_{31} = p_{31}/w_{31} = \rho_{21}(0)$.

For the longitudinal velocity we obtain the linear equation

$$\frac{\partial}{\partial\eta}\left(w_{31}\frac{\partial v_{x31}}{\partial\eta}\right) + \frac{1}{\eta} w_{31}\frac{\partial v_{x31}}{\partial\eta} + \frac{1}{\eta^2}\frac{\partial}{\partial\varphi}\left(w_{31}\frac{\partial v_{x31}}{\partial\varphi}\right) = 0$$

from which, introducing the variable $\eta_1$ we obtain the Laplace equation. Since the limit conditions presuppose the absence of singularity in $v_{x31}$ in all of the plane $\eta_1, \varphi$ and the independence of $x$, we conclude that $v_{x31} = v_{x21}(0)$.

Projections of the equations of motion on the axes $r$ and $\varphi$ in the first approximation enabled us to determine the pressure. However the esimate with respect to $x$ of the subsequent terms in the pressure expansion (2.1) allows the use of these equations once more, Taking also into account the equation of continuity, we obtain for $v_{r31}$ and $v_{\varphi 31}$ the system

$$-1 + \frac{\partial v_{r31}}{\partial\eta} + \frac{v_{r31}}{\eta} + \frac{1}{\eta}\frac{\partial v_{\varphi 31}}{\partial\varphi} = 0 \tag{2.4}$$
$$\frac{2}{3}\frac{\partial}{\partial\eta}\left(2\frac{\partial v_{r31}}{\partial\eta} - \frac{v_{r31}}{\eta} - \frac{1}{\eta}\frac{\partial v_{\varphi 31}}{\partial\varphi}\right) + \frac{2}{\eta}\left(\frac{\partial v_{r31}}{\partial\eta} - \frac{v_{r31}}{\eta} - \right.$$

$$\frac{1}{\eta}\frac{\partial v_{\varphi 31}}{\partial \varphi}\Big) + \frac{1}{\eta}\frac{\partial}{\partial \varphi}\left(\frac{1}{\eta}\frac{\partial v_{r31}}{\partial \varphi} + \frac{\partial v_{\varphi 31}}{\partial \eta} - \frac{v_{\varphi 31}}{\eta}\right) = 0$$

$$\frac{\partial}{\partial \eta}\left(\frac{1}{\eta}\frac{\partial v_{r31}}{\partial \varphi} + \frac{\partial v_{\varphi 31}}{\partial \eta} - \frac{v_{\varphi 31}}{\eta}\right) + \frac{1}{\eta}\frac{\partial}{\partial \varphi}\left[\frac{4}{3}\left(\frac{1}{\eta}\frac{\partial v_{\varphi 31}}{\partial \varphi} + \frac{v_{r31}}{\eta}\right) - \frac{2}{3}\frac{\partial v_{r31}}{\partial \eta}\right] + \frac{2}{\eta}\left(\frac{1}{\eta}\frac{\partial v_{r31}}{\partial \varphi} + \frac{\partial v_{\varphi 31}}{\partial \eta} - \frac{v_{\varphi 31}}{\eta}\right) = 0$$

that consists of three equations for the determination of two functions $v_{r31}$ and $v_{\varphi 31}$, and is consequently **overdetermined.**

Let us consider the question of consistency of this system. Using the first equation we eliminate in the second the function $v_{\varphi 31}$ and obtain

$$\eta\frac{\partial^2 v_{r31}}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial^2 v_{r31}}{\partial \varphi^2} + 3\frac{\partial v_{r31}}{\partial \eta} + \frac{v_{r31}}{\eta} - 2 = 0 \tag{2.5}$$

Eliminating in the third equation of the system (2.4) function $v_{\varphi 31}$, using the first equation, we obtain

$$\frac{\partial}{\partial \eta}\left\{\eta\frac{\partial^2 v_{r31}}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial^2 v_{r31}}{\partial \varphi^2} + 3\frac{\partial v_{r31}}{\partial \eta} + \frac{v_{r31}}{\eta}\right\} = 0 \tag{2.6}$$

Selecting the constant of integration equal two, so that (2.5) and (2.6) coincide we conclude that $v_{31}$ is to be subordinated to Eq.(2.5) and obtain $v_{\varphi 31}$ from the first equation of system (2.4). Passing to the variable $\eta_1 = \ln\eta$ and introducing the new function

$$\overset{\circ}{v}_{r31} = -{}^1/_2\exp(2\eta_1) + \exp(\eta_1)v_{r31}$$

we obtain for $\overset{\circ}{v}_{r31}$ the Laplace equation. Function $\overset{\circ}{v}_{r31}$ is determined in the bend $-\infty < \eta_1 < \infty$, $0 \leqslant \varphi \leqslant 2\pi$, and the condition

$$\overset{\circ}{v}_{r31}(x, \eta_1, 0) = \overset{\circ}{v}_{r31}(x, \eta_1, 2\pi), \quad \frac{\partial \overset{\circ}{v}_{r31}(x, \eta_1, 0)}{\partial \varphi} = \frac{\partial \overset{\circ}{v}_{r31}(x, \eta_1, 2\pi)}{\partial \varphi} \tag{2.7}$$

of periodicity with respect to $\varphi$ must be satisfied.

Condition (2.2) as $\eta \to \infty$ and that of boundedness as $\eta \to 0$ imposed on function $v_{r31}$ enables us to write for $\overset{\circ}{v}_{r31}$

$$\overset{\circ}{v}_{r31} = -B\exp(\eta_1)\cos\varphi + o(\exp(\eta_1)), \quad \eta_1 \to \infty \tag{2.8}$$
$$\overset{\circ}{v}_{r31} = O(\exp(\eta_1)), \quad \eta_1 \to -\infty$$

Using for the construction of function $\overset{\circ}{v}_{r31}$ the method of Fourier, we conclude that the solution of the Laplace equation that satisfies conditions (2.7) and (2.8) is unique and has the form

$$\overset{\circ}{v}_{r31} = -B\exp(\eta_1)\cos\varphi$$

Reverting to input functions (2.1), we find that they are exactly equal to their limit values (2.2). The determination of parameters of flow in the subtrail is completed.

The problem considered here does not admit any other steady solutions, which in view of its very specific formulation is not obvious a priori .

For a clearer idea of gas motion in the subtrail we pass to a Cartesian system of coordinates $(y, z)$ and write the projections of velocity onto these axes

$$v_y = \frac{1}{2(\varkappa+1)}\frac{y}{x} - \frac{1}{\varkappa+1}b^{1/2}x^{-(2+\varkappa)/2(\varkappa+1)}B, \quad v_z = \frac{1}{2(\varkappa+1)}\frac{z}{x} \tag{2.9}$$

It follows from (2.9) that, if we introduce a new independent variable

$$y_c = y - 2b^{1/2}x^{\varkappa/2(\varkappa+1)}B \tag{2.10}$$

and pass from the new Cartesian system of coordinates $(y_c, z)$ to a polar system $(r_c, \varphi_c)$, then the projections of the velocity vector on the axes $r_c$ and $\varphi_c$ assume the form

$$v_{rc} = \frac{1}{2(\varkappa+1)}r_c, \quad v_{\varphi c} = 0 \tag{2.11}$$

Hence, in conformity with (2.11), the flow in the transverse plane of the subtrail has a central symmetry about point $y_c = 0, z = 0$. At that point the transverse velocity vector vanishes. Tracing the position of this point in the original Cartesian system of coordinates $(x, y, z)$, we find that in conformity with (2.10) it performs oscillations in the plane $z=0$ with increasing amplitude as $x$ increases

$$y = 2b^{1/2}b_y x^{\varkappa/2(\varkappa+1)}\,[C_1 \cos(k_3 \ln x) + C_2 \sin(k_3 \ln x)]$$

The direction field is much simpler in the transverse plane of the subtrail, and is the field of velocities of a source with center at $y_c = 0, z = 0$. The longitudinal component of the vortex vector $\omega_x$ vanishes in this approximation.

We note in conclusion that the formation in the transverse plane of vortex zones associated with the lift force is a common occurence for three-dimensional flows. Thus, in the subsonic flow over bodies subjected to the lift force, past of the body in the region of the laminar flow two vortices are formed which rotate in the opposite directions /7/. The pattern in hypersonic flow is much more complex, the presence of the lift force leads to the occurence of vortices converging to the center.

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